October 15, 2024

Handout for Meeting 6

Logical Expressivism and Expressivist Logic

I. Reasons and Logic: Logicism and Expressivism

a) Logicism about the relations between *reasons* and *logic* is the view that good reasons are, in the end, always *logically* good reasons, articulated by deductive logical relations of implication and inconsistency.
b) Expressivism about the relations between *reasons* and *logic* is the view that the distinctive task of logical vocabulary is an *expressive* one, to make explicit in claimable form antecedent material reason relations of implication and incompatibility. The principle job of logic is to let us *say* something, rather than to let us *prove* something.
c) In the philosophy of logic, this is an answer to the *demarcation* question: what makes something a logical concept, or a bit of logical vocabulary? The *correctness* question (which is the right logic?) then lapses.
d) Logical vocabulary as extending a base vocabulary, rather than as a free-standing vocabulary. *Applied* logic is conceptually prior to, and more important than, the degenerate case of *pure* logic.
e) Expressivist criteria of demarcation/adequacy:

The reason relations governing sentences containing logical vocabulary must be

i) ***El***aborated from and
ii) ***Ex***plicative of

(Shorthand: "**LX** for") the *reason relations* of a *non*logical (material) *base* vocabulary.

For L: Must be able to *conservatively* extend the base vocabulary by *computing* the reason relations of the logically extended vocabulary from the reason relations of the underlying material base vocabulary.

For X: Paradigm of what counts as making reason relations explicit as claimables: DD and II (for genuinely expressive, as opposed to merely aggregative connectives):

• Deduction-Detachment (DD) Condition on Conditionals:
Γ |~ A→B if and only if Γ,A |~ B.
• Incoherence-Incompatibility (II) Condition on Negation:
Γ |~ ¬A if and only if Γ#A, i.e. **Γ,A |~ .**

f) Substructural reason relations—those not topologically closed or explicitation closed—present a challenge *both* for logicism *and* for expressivism. If logical reason relations are structurally closed, how can they either underlie or express open reason relations?
g) Ideal: A logic that is *universally* LX, LX for *any* and *every* base vocabulary (with CO).

II. Sequent Calculus Metavocabularies for Expressivist Logic: Gentzen and NMMS

LK (Gentzen):

L→: $$\frac{\Gamma \mathrel{|\!\sim} \Delta, A \quad \Gamma, B \mathrel{|\!\sim} \Delta}{\Gamma, A \rightarrow B \mathrel{|\!\sim} \Delta}$$

R→: $$\frac{\Gamma, A \mathrel{|\!\sim} \Delta, B}{\Gamma \mathrel{|\!\sim} \Delta, A \rightarrow B}$$

L¬: $$\frac{\Gamma \mathrel{|\!\sim} \Delta, A}{\Gamma, \neg A \mathrel{|\!\sim} \Delta}$$

R¬: $$\frac{\Gamma, A \mathrel{|\!\sim} \Delta}{\Gamma \mathrel{|\!\sim} \Delta, \neg A}$$

L∧: $$\frac{\Gamma, A, B \mathrel{|\!\sim} \Delta}{\Gamma, A \wedge B \mathrel{|\!\sim} \Delta}$$

R∧: $$\frac{\Gamma \mathrel{|\!\sim} \Delta, A \quad \Gamma \mathrel{|\!\sim} \Delta, B}{\Gamma \mathrel{|\!\sim} \Delta, A \wedge B}$$

L∨: $$\frac{\Gamma, A \mathrel{|\!\sim} \Delta \quad \Gamma, B \mathrel{|\!\sim} \Delta}{\Gamma, A \vee B \mathrel{|\!\sim} \Delta}$$

R∨: $$\frac{\Gamma, \mathrel{|\!\sim} \Delta, A, B}{\Gamma \mathrel{|\!\sim} \Delta, A \vee B}$$

NMMS (NonMonotonic MultiSuccedent logic, after Ketonen):

L→: $$\begin{array}{c} \Gamma \mathrel{|\!\sim} \Delta, A \quad \Gamma, B \mathrel{|\!\sim} \Delta \quad \Gamma, B \mathrel{|\!\sim} \Delta, A \\ \hline\hline \Gamma, A \rightarrow B \mathrel{|\!\sim} \Delta \end{array}$$

R→: $$\begin{array}{c} \Gamma, A \mathrel{|\!\sim} \Delta, B \\ \hline\hline \Gamma \mathrel{|\!\sim} \Delta, A \rightarrow B \end{array}$$

L¬: $$\begin{array}{c} \Gamma \mathrel{|\!\sim} \Delta, A \\ \hline\hline \Gamma, \neg A \mathrel{|\!\sim} \Delta \end{array}$$

R¬: $$\begin{array}{c} \Gamma, A \mathrel{|\!\sim} \Delta \\ \hline\hline \Gamma \mathrel{|\!\sim} \Delta, \neg A \end{array}$$

L∧: $$\begin{array}{c} \Gamma, A, B \mathrel{|\!\sim} \Delta \\ \hline\hline \Gamma, A \wedge B \mathrel{|\!\sim} \Delta \end{array}$$

R∧: $$\begin{array}{c} \Gamma \mathrel{|\!\sim} \Delta, A \quad \Gamma \mathrel{|\!\sim} \Delta, B \quad \Gamma \mathrel{|\!\sim} \Delta, A, B \\ \hline\hline \Gamma \mathrel{|\!\sim} \Delta, A \wedge B \end{array}$$

L∨: $$\begin{array}{c} \Gamma, A \mathrel{|\!\sim} \Delta \quad \Gamma, B \mathrel{|\!\sim} \Delta \quad \Gamma, A, B \mathrel{|\!\sim} \Delta \\ \hline\hline \Gamma, A \vee B \mathrel{|\!\sim} \Delta \end{array}$$

R∨: $$\begin{array}{c} \Gamma, \mathrel{|\!\sim} \Delta, A, B \\ \hline\hline \Gamma \mathrel{|\!\sim} \Delta, A \vee B \end{array}$$

Two metainferential consequence relations among sequents: *derivability* and *admissibility*. Double-line rules are reversible, preserving admissibility.

Fact: **NMMS is *universally* LX for all base vocabularies that satisfy CO.**

Kaplan's expressive completeness representation theorem for NMMS (*RLLR* p. 130):
For any set *AtomicImp* of sequents in any base vocabulary L_B, there is a nonempty set of sequents *ExtImp* in the logical extension of L_B by NMMS such that every individual sequent in *ExtImp* is derivable if and only if all the sequents of *AtomicImp* hold. And conversely, for any set of sequents *ExtImp* in the logical extension of L_B by NMMS, we can compute the exact set *AtomicImp* of sequents defined on the lexicon of the base vocabulary that must hold in order for all the sequents in *ExtImp* to be derivable.

Explicitly Marking Local Regions of Structure (*RLLR* section 3.3 pp. 131ff):
Can mark implications that hold *persistently* (monotonically), so that ∀X⊆L[Γ,X|~A,Δ], by "$\Gamma|\!\sim^{\uparrow}A,\Delta$." Then can introduce an operator to codify the persistence of that sequent as "Γ|~□A,Δ." Similarly, we can make explicit local regions of *classicality* (MO + CT).

III. Trilogics K3 and LP from an Expressivist Perspective

In the standard multi-valued semantic setting, these are the Strong Kleene matrices (multivalued truth tables) for connectives, common to both the logics K3 (Kripke) and LP (Priest):

¬	
1	0
½	½
0	1

⊃	**1**	½	**0**
1	1	½	0
½	1	½	½
0	1	1	1

∧	**1**	½	**0**
1	1	½	0
½	½	½	0
0	0	0	0

∨	**1**	½	**0**
1	1	1	1
½	1	½	½
0	1	½	1

K3 treats only 1 as designated (preserved by good implications).
LP treats both 1 and ½ as designated (preserved by good implications).
Thereby, K3 interprets ½ as "Neither", and LP interprets ½ as "Both."
K3 is the paracomplete logic of truth-value *gaps*.
LP is the paraconsistent logic of truth-value *gluts*.

Construing the trilogics from an inferential point of view, in the presence of Cut (CT), and Monotonicity (MO), for arbitrary sets of sentences Γ and Δ, **A|~B** licenses:

i) Premissory substitution license:

$$\frac{\Gamma, B|\!\sim\Delta}{\Gamma, A|\!\sim\Delta}$$

ii) Conclusory substitution license:

$$\frac{\Gamma|\!\sim A, \Delta}{\Gamma|\!\sim B, \Delta}$$

K3's implication-codifying turnstile is of type (i), having the *meta*inferential significance of a *premissory* substitution license.
LP's implication-codifying turnstile is of type (ii), having the *meta*inferential significance of a *conclusory* substitution license.

Q: Why does the logic of premissory substitution metainferences also show up as the paracomplete logic of truth-value *gaps* and the logic of conclusory substitution metainferences also show up as the paraconsistent logic of truth-value *gluts*?
A: In the underlying substructural |~ relation in NMMS, it can happen that in the base vocabulary, for some sentences A and B transitivity fails in such a way that one cannot Cut on A∨¬A as a premise, or B∧¬B as a conclusion. In this sense, the logic of premissory substitutional metainferences does not *accept* some instances of Excluded Middle and the logic of conclusory substitutional metainferences does not *reject* some instances of Noncontradiction. In a semantic setting that construes consequence in terms of preservation of designated truth-values, these facts show up as gaps and gluts. (Cf. *RLLR* pp. 244-245).
K3 and LP are both fully transitive and monotonic.

Four examples of distinctions-with-relations emerging from relaxation of structure:

1. (Well-known:) One of Gentzen's astonishing accomplishments was to show that the very same set of connective definitions and structural constraints (MO, CT) specifies *intuitionistic* logic if one requires that sequents only have single formulae on the right ("single-succedent" sequents), and specifies *classical* logic if one relaxes that requirement and allows more than one formula on the right ("multi-succedent" sequents).

2. (New:) One large lesson of this week is that connective-defining sequent rules that specify the *same* logic under strong structural closure constraints (MO, CT), can come apart and determine different consequence relations and incompatibilities when those structural constraints are relaxed. Gentzen's LK rules and Ketonen's reversible rules both specify *classical* logic if MO and CT hold.

3. (Well-known:) Strong Kleene multivalue truth tables or matrices yield very different logics if they are conjoined with different notions of consequence. K3 and LP are alike in using preservation-of-designatedness of multivalues to define consequence, differing only in which multivalues are designated. The Strict-Tolerant logic ST shows a quite different way to define consequence.

4. (New:) We ended by comparing definitions of consequence *salva veritate* (which includes K3 and LP) vs. *salva consequentia*. Under fully closed structural conditions, if A|~B, then A can be substituted everywhere for B as a premise, and B can be substituted everywhere for A as a conclusion, saving the goodness of sequents. Defining consequence by considering substitutions *salva consequentia* (which substitutions of sentences for sentences preserves the goodness of reason relations) splits into two different metainferential relations, *premissory*-role inclusion and *conclusory*-role inclusion, if mixed context Cut (transitivity) fails.